# LightPipes for Mathcad

Reconstruction of the Phase from measured intensity distributions..

Suppose we have measured two intensity distributions of a beam with for example a ccd camera: one in the near field and one after propagation of a distance z. The two intensity distributions have been stored on disk:

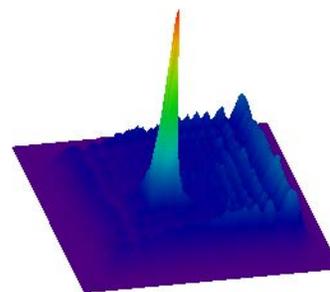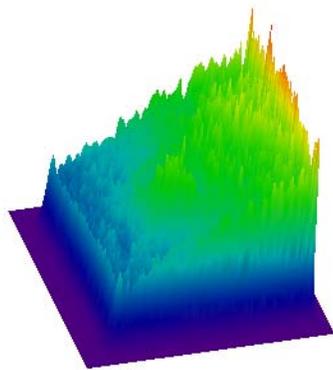$I_{near} := READ\_IMAGE("lightpipes-phaserecovery-near-field.jpg")$

$I_{far} := READ\_IMAGE("lightpipes-phaserecovery-far-field.jpg")$
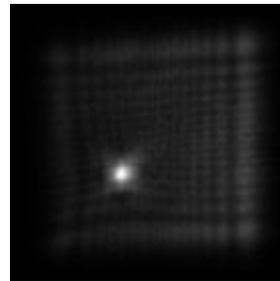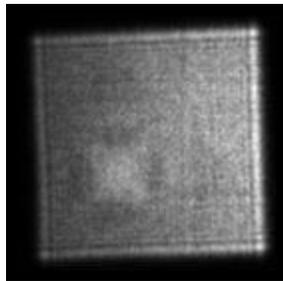
$size \equiv 11 \cdot mm$

$\lambda \equiv 632.8 \cdot nm$

$z \equiv 2 \cdot m$

I_{near}

I_{far}

Measured near field intensity

Measured far field intensity after propagation of z=2m.

$$N := \text{rows}\left(I_{near}\right)$$

$$N = 140 \qquad \text{cols}\left(I_{near}\right) = 140$$

The knowledge of these intensity patterns is sufficient to reconstruct the phase of the beam. To do this we repeatedly substitute the far field intensity in an arbitrary (uniform) field, propagate this field back to the near field position, substitute the near field intensity and propagate to the far field. After sufficient iterations the phase and the intensity of the beam become stable after each iteration and the phase distribution has been recovered.

The iteration for reconstruction of the phase pattern goes as follows:

Number of iterations: $\quad n \equiv 1000$

Dimension of the bigger grid, necessary for LPForvard: $\quad size_{new} \equiv 20 \cdot mm \qquad N_{new} \equiv 256$

$$A := \left| \begin{array}{l} K \leftarrow \text{LPBegin}\left(\dfrac{size}{m}, \dfrac{\lambda}{m}, N\right) \\[2mm] \text{for } k \in 0..n \\[1mm] \qquad \left| \begin{array}{l} K \leftarrow \text{LPSubIntensity}\left(I_{far}, K\right) \\[2mm] K \leftarrow \text{LPInterpol}\left(\dfrac{size_{new}}{m}, N_{new}, 0, 0, 0, 1, K\right) \\[2mm] K \leftarrow \text{LPForvard}\left(-\dfrac{z}{m}, K\right) \\[2mm] K \leftarrow \text{LPInterpol}\left(\dfrac{size}{m}, N, 0, 0, 0, 1, K\right) \\[2mm] K \leftarrow \text{LPSubIntensity}\left(I_{near}, K\right) \\[2mm] K \leftarrow \text{LPForvard}\left(\dfrac{z}{m}, K\right) \\[2mm] Out_k \leftarrow K \end{array} \right. \\[2mm] Out \end{array} \right.$$

The iteration starts with a uniform field.

Here the measured far field intensity is substituted.

Interpolation to a bigger grid to make LPForvard happy.

Back propagation of the far field to the near field plane.
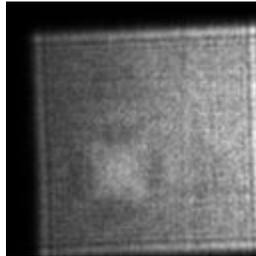
Interpolation to the original grid.

Substitution of the measured near field pattern.

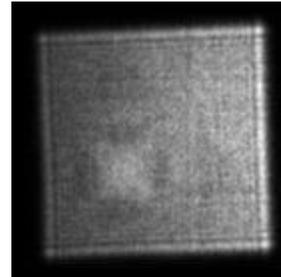Propagation of the near field to the far field plane.

Fill the output structure.

The reconstructed near field after n iterations is:
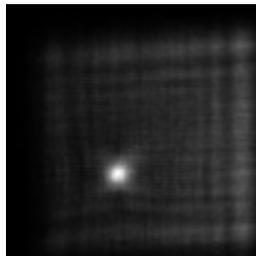$$F_{nearrec} := LPForvard\left(-\frac{z}{m}, A_n\right)$$

$$I_{nearrec} := LPIntensity(2, F_{nearrec})$$

$I_{nearrec}$

Reconstructed near field
intensity, $I_{nearrec}$.

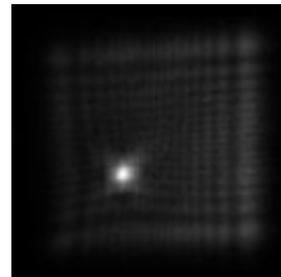I_{near}

Original measured near
field intensity, I_{near}.

The far reconstructed far field is calculated by:
$$F_{farrec} := LPForvard\left(\frac{z}{m}, F_{nearrec}\right)$$

$$I_{farrec} := LPIntensity(2, F_{farrec})$$

I_{farrec}

Reconstructed far field
intensity, I_{farrec}.
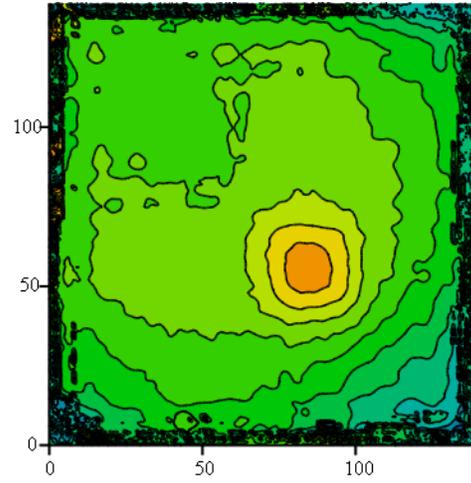

I_{far}

Original measured far
field intensity, I_{far}.
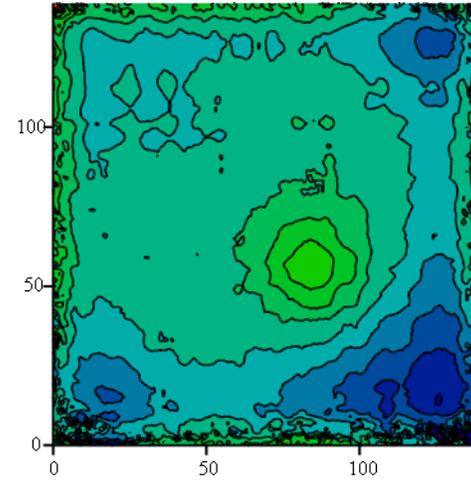
The reconstructed unwrapped phase patterns of the near and far fields are:

$\Phi_{nearrec} := LPPhase(F_{nearrec})$ $\qquad$ $\Phi_{nearrec} := LPPhaseUnwrap(1, \Phi_{nearrec})$

$\Phi_{farrec} := LPPhase(F_{farrec})$ $\qquad$ $\Phi_{farrec} := LPPhaseUnwrap(1, \Phi_{farrec})$

$\Phi_{nearrec}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Φ_{farrec}

The reconstructed phase patterns of the near and far fields.